

Rotating QCD string and the meson spectrum

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Abstract

The spectra of light–light and heavy–light mesons are described by spinless Salpeter equation and Dirac equation respectively, which predict linear dependence of the meson mass squared M^2 on angular momentum J and number of radial nodes n . Both spectra are computed by the WKB method and shown to agree with exact numerical data within few percent even for the lowest levels. The drawback of Salpeter and Dirac equation is that (inverse) Regge slopes do not coincide with the string ones, $2\pi\sigma$ and $\pi\sigma$ respectively, because the string dynamics is not taken into account properly. The lacking string rotation is introduced via effective Hamiltonian derived from QCD which generates linear Regge trajectories for light mesons with the correct string slope.

1 Introduction

QCD is believed to be the fundamental theory of strong interactions and the meson spectroscopy is to be derived from QCD. The spectrum of mesons has been treated in a sequence of models [1, 2] which may be called QCD motivated, but still not directly derived from the QCD Lagrangian.

The problem of the celebrated Regge behaviour of the hadron spectra has been discussed in literature not once (see *e.g.* [3] and references herein) but still attracts considerable attention. The light–light meson spectrum obtained so far and reasonably describing experiment can be written in the form

$$M_{ll}^2(n, J) = (c_n^{(ll)} n + c_J^{(ll)} J + \Delta M_p^2 + \Delta M_s^2), \quad (1)$$

where n is the radial quantum number, J being the total angular momentum, ΔM_p^2 contains the perimeter (self-energy) mass correction as well as corrections to the first two terms, while ΔM_s^2 takes into account spin splittings.

For heavy–light mesons a similar relation holds true with the subscript ll changed for hl in all coefficients:

$$M_{hl}^2(n, J) = (c_n^{(hl)} n + c_J^{(hl)} J + \Delta M_p^2 + \Delta M_s^2). \quad (2)$$

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From general physical considerations one expects the spectra (1) and (2) to follow from a string-like picture of confinement which predicts the (inverse) Regge slope

$$c_J^{(l)} = 2\pi\sigma, \quad c_J^{(hl)} = \pi\sigma. \quad (3)$$

Additional daughter Regge trajectories are given by vibrational excitations missing in (1) and (2), which are due to hybrid excitations, *i.e.* constituent gluons attached to the fundamental string [4]. In what follows we are interested only in radial and orbital excitations of the string.

The string slope (3) is an important criterion and check for any QCD inspired model since it requires the correct account for the rotation of the string, which is not present in the potential models considered so far. For example, relativistic spinless Salpeter equation with confinement reduced to the linearly rising potential between quarks yields

$$c_J^{(l)}(\textit{Salpeter}) = 8\sigma, \quad c_J^{(hl)}(\textit{Salpeter}) = 4\sigma \quad (4)$$

that is about 25% larger than (3), whereas the one-body Dirac equation with the linearly rising potential leads to

$$c_J^{(hl)}(\textit{Dirac}) = 4\sigma, \quad (5)$$

if the potential is added to the energy term (vector confinement¹), and

$$c_J^{(hl)}(\textit{Dirac}) = 2\sigma, \quad (6)$$

for the potential added to the mass term (scalar confinement). Both results lead to considerable discrepancies with (3) and, as will be shown later, this happens because the rotation of the string, and hence momentum dependence of the effective potential, is not taken into account.

It was found few years ago [5] that starting from the area law for Wilson loops one arrives at the relativistic Hamiltonian for the spinless quark and antiquark which possesses two different regimes: potential regime for small angular momenta L and any n , and string-like one for large L and fixed n . In the latter case the dominant term in the Hamiltonian indeed describes the rotating QCD string, so that the string Regge slope (3) is readily reproduced. Similar results were obtained independently by numerical analysis of the spinless quark–antiquark system [6].

In the present paper we concentrate on the quasiclassical approach to mesons, as the WKB method allows to obtain analytic formulae for the meson spectra of surprisingly high accuracy thus giving evidence for the quasiclassical dynamics of confined quarks in the meson.

Therefore our first task will be to check the accuracy of the WKB approximation for those cases where exact solutions are feasible: spinless Salpeter equation for light–light mesons (potential regime of the general QCD string formalism [5]) and the Dirac equation for linear confining potential for the case of heavy–light system. We argue that the accuracy of WKB results is very good even for lowest states. However the slopes in both cases are incorrect, as in (4) and (6) respectively.

At this point we come to the main purpose of this study — to include the proper string dynamics, whereby abandoning the notion of local potential and introducing a new entity,

¹Here we leave aside the well-known problem of the Klein paradox revealing itself in case of vector confinement.

the QCD string, the effect which can not be recasted in terms of local potential. We use the Hamiltonian derived in [5] and calculate the quasiclassical spectrum of light mesons. The results represent celebrated straight-line Regge trajectories even for low-lying states with the slope very close to the expected string slope (3).

In conclusion we demonstrate how other effects (spin and colour Coulomb interaction) can be included in the same Hamiltonian to make a direct comparison with experiment.

2 Meson spectrum and quasiclassical approximation

We start with the spinless Salpeter equation which describes relativistic quark and antiquark of equal masses m with angular momentum $l = 0$ and spin effects neglected (see [5] for the derivation of this equation from the general meson Green function in QCD).

$$(2\sqrt{p_r^2 + m^2} + \sigma r)\psi_n = M_n^{(ll)}\psi_n \quad (7)$$

The Bohr–Sommerfeld condition looks like

$$\int_0^{r_+} p_r(r)dr = \pi \left(n + \frac{3}{4} \right), \quad n = 0, 1, 2, \dots, \quad r_+ = \frac{M_n^{(ll)} - 2m}{\sigma} \quad (8)$$

that yields

$$M_n^{(ll)} \sqrt{(M_n^{(ll)})^2 - 4m^2} - 4m^2 \ln \frac{\sqrt{(M_n^{(ll)})^2 - 4m^2} + M_n^{(ll)}}{2m} = 4\sigma\pi \left(n + \frac{3}{4} \right) \quad (9)$$

A similar consideration for the heavy–light system of masses m and M ($M \rightarrow \infty$) gives

$$(\sqrt{p_r^2 + m^2} + \sigma r)\psi_n = M_n^{(hl)}\psi_n \quad (10)$$

$$M_n^{(hl)} \sqrt{(M_n^{(hl)})^2 - m^2} - m^2 \ln \frac{\sqrt{(M_n^{(hl)})^2 - m^2} + M_n^{(hl)}}{m} = 2\sigma\pi \left(n + \frac{3}{4} \right). \quad (11)$$

Accuracy of WKB approximation (9), (11) can be tested *vs* exact solutions of the Salpeter equations (recently accuracy of WKB approximation was checked for light–light mesons in [7]). In Table 1 this comparison is given for the light–light system with $m = 0$ and heavy–light one with $m_q = 0.01 \text{ GeV}$ and $M_{\bar{q}} = 10 \text{ GeV}$. The mass $M_n^{(hl)}$ in the latter case actually refers to the difference of the total mass of the heavy–light system and the mass of the heavy antiquark.

Summarizing, one can say that spectra (9), (11) (as function of n for $l = 0$) indeed have the form (1), (2) with the corrections at large n in the form

$$\Delta M^2 = O \left(\frac{m^2}{M_n^2} \ln \frac{M_n}{m} \right) = O \left(\frac{\ln n}{n} \right). \quad (12)$$

The WKB spectrum is linear in n and its accuracy is about 3-4% even for the lowest state.

n	0	1	2	3	4	5
$M_n^{(ll)}(\text{WKB})$	1.373	2.097	2.629	3.070	3.455	3.802
$M_n^{(ll)}(\text{exact})$	1.412	2.106	2.634	3.073	3.457	3.803
$M_n^{(hl)}(\text{WKB})$	0.971	1.483	1.859	2.171	2.443	2.688
$M_n^{(hl)}(\text{exact})$	1.014	1.524	1.917	2.246	2.537	2.800

Table 1: Comparison of the WKB and exact spectra for Salpeter equations (7) for massless quark and antiquark and (10) for the quark mass $m_q = 0.01 \text{ GeV}$ and antiquark mass $M_{\bar{q}} = 10 \text{ GeV}$. In both cases $\sigma = 0.2 \text{ GeV}^2$ and $l = 0$.

We now turn to the case of the Dirac equation with linear confining potential studied in [8]. The WKB method for the Dirac equation was thoroughly investigated in [9] and recently applied to the case of confining potential [10]. Let us briefly recall the results here.

The Dirac equation with scalar (U) and vector (V) local potentials has the form

$$(\vec{\alpha}\vec{p} + \beta(m + U) + V)\psi_n = \varepsilon_n\psi_n, \quad (13)$$

and the WKB quantization condition is [9]

$$\int_{r_-}^{r_+} \left(p + \frac{\kappa w}{pr} \right) dr = \pi \left(n + \frac{1}{2} \right), \quad n = 0, 1, 2, \dots, \quad (14)$$

where

$$\begin{aligned} p &= \sqrt{(\varepsilon - V)^2 - \frac{\kappa^2}{r^2} - (m + U)^2}, \\ w &= -\frac{1}{2r} - \frac{1}{2} \frac{U' - V'}{m + U + \varepsilon - V}, \\ |\kappa| &= j + \frac{1}{2} \end{aligned} \quad (15)$$

An approximate quasiclassical solution of (13) obtained in [10] for the case $m = 0$, $V = 0$, $U = \sigma r$ is

$$\varepsilon_n^2 = 2\sigma \left(2n + j + \frac{3}{2} + \frac{\text{sgn}\kappa}{2} + \frac{\kappa\sigma}{\pi\varepsilon_n^2} \left(0.38 + \ln \frac{\varepsilon_n^2}{\sigma|\kappa|} \right) + O \left(\left(\frac{\kappa\sigma}{\varepsilon_n^2} \right)^2 \right) \right). \quad (16)$$

The last two terms on the r.h.s. of equation (16) are sub-leading for large n and are generated by the term $\frac{\kappa w}{pr}$ (see (14)). One can see that the (inverse) Regge slope in j in (16) is equal to 2σ coinciding with the exact result (6), but is not of string type. As it was expected a j -independent scalar potential does not describe the physical phenomenon of rotating string.

Still the accuracy of the WKB approximation is impressing. In Table 2 one can see the comparison of exact eigenvalues computed in [8] with quasiclassical ones and with those obtained from (16). The discrepancy is less than 1% even for the lowest state and it is much better for higher states.

n	l	j	κ	M_{exact}	M_{WKB}	$M_{\text{eq.(16)}}$
0	2	3/2	2	1.209	1.209	1.208
0	1	1/2	1	1.024	1.025	1.025
0	0	1/2	-1	0.725	0.726	0.733
0	1	3/2	-2	0.957	0.960	0.966
0	2	5/2	-3	1.149	1.149	1.155

Table 2: Comparison of the exact spectrum for Dirac equation (13) with WKB spectrum and approximate one given by equation (16) for $m = 0$, $V = 0$ and $U = \sigma r$ with $\sigma = 0.2 \text{ GeV}^2$.

3 Rotating string in the spinless quark Hamiltonian

Let us turn back to the spinless Salpeter equation for the light–light meson and take the non-zero angular momentum into account. As it was shown above the Salpeter equation with local l -independent potential leads to the incorrect Regge slope (4), and therefore this case requires a special treatment. One needs a Hamiltonian taking into account dynamical degrees of freedom of the string, *e.g.* in the form of time derivatives of string coordinates. This was done explicitly in [5], where it was shown that starting from the QCD Lagrangian and writing the gauge invariant $q\bar{q}$ Green function for confined spinless quarks in the Feynman-Schwinger representation, one can arrive at the Lagrange function of the system in the well-known form

$$L(\tau) = -m_1\sqrt{\dot{x}_1^2} - m_2\sqrt{\dot{x}_2^2} - \sigma \int_0^1 d\beta \sqrt{(\dot{w}w')^2 - \dot{w}^2 w'^2}, \quad (17)$$

where τ denotes the proper time of the system, the first two terms stand for quarks, whereas the last one describes the minimal string with tension σ developed between the constituents; $w_\mu(\tau, \beta)$ being the string coordinate. Adopting the straight-line ansatz for the minimal string, *i.e.* $w_\mu(\tau, \beta) = \beta x_{1\mu} + (1 - \beta)x_{2\mu}$, synchronizing the quarks proper times, $x_{10} = x_{20} = \tau = t_{\text{lab}}$ and introducing auxiliary fields to get rid of the square roots (see *e.g.* [11]) one can obtain the following Hamiltonian in the centre of mass frame (we consider the case of equal masses m)

$$H = \frac{p_r^2 + m^2}{\mu(\tau)} + \mu(\tau) + \frac{\hat{L}^2/r^2}{\mu + 2 \int_0^1 (\beta - \frac{1}{2})^2 \nu(\beta) d\beta} + \frac{\sigma^2 r^2}{2} \int_0^1 \frac{d\beta}{\nu(\beta)} + \int_0^1 \frac{\nu(\beta)}{2} d\beta, \quad (18)$$

where the two auxiliary positive functions $\mu(\tau)$ and $\nu(\beta, \tau) \equiv \nu(\beta)$ are to be varied and to be found from the minimum of H yielding quark energy and string energy density respectively. A more detailed analysis of the role played by auxiliary fields can be found in *e.g.* [12].

Note that Hamiltonian (18) has a form of the sum of “kinetic” and “potential” terms only due to auxiliary fields μ and ν . If one gets rid of them by substituting their extremal values, the resulting Hamiltonian possesses a very complicated form which makes its analysis and quantization hardly possible.

The centrifugal potential in Hamiltonian (18) is of special interest to us and, most of all, the second term in the denominator. It is this term that describes extra inertia due to the string connecting the quarks. Neglecting this term and taking extrema in the auxiliary fields one easily arrives at the ordinary Salpeter Hamiltonian with linearly rising potential,

whereas account for this extra term describes the proper string rotation and brings the slope of the Regge trajectory into correct form (3). In the nonrelativistic expansion of Hamiltonian (18) this term yields the so-called string correction to the leading confining potential σr [5]

$$\Delta H_l = -\frac{\sigma \hat{L}^2}{6m^2 r},$$

the part of the interaction which explicitly depends on the angular momentum.

Hamiltonian (18) assumes especially simple form in the case of zero angular momentum and after excluding the auxiliary fields produces Salpeter equation (7).

Variation of (18) over $\nu(\beta)$ gives the stationary energy distribution along the string with β ($0 \leq \beta \leq 1$) being the coordinate along the string. Thus one obtains

$$\nu_0(\beta) = \frac{\sigma r}{\sqrt{1 - 4y^2(\beta - \frac{1}{2})^2}}, \quad (19)$$

where y is to be found from the transcendental equation

$$\frac{\hat{L}}{\sigma r^2} = \frac{1}{4y^2}(\arcsin y - y\sqrt{1 - y^2}) + \frac{\mu y}{\sigma r}, \quad (20)$$

and $\hat{L}^2 = l(l + 1)$.

Note that the maximal possible value of y , $y = 1$, yields the energy distribution $\nu_0^{free}(\beta)$ corresponding to the free open string (string without quarks at the ends) [5, 6].

In the general case inserting the extremal function $\nu_0(\beta)$ one obtains from (18)

$$H = \frac{p_r^2 + m^2}{\mu(\tau)} + \mu(\tau) + \frac{\sigma r}{y} \arcsin y + \mu(\tau)y^2 \quad (21)$$

with $y = y(\hat{L}, r, \mu)$ defined by equation (20). Unfortunately no rigorous analytic calculations are possible anymore, so one has to rely upon numerical calculations. But let us first perform some analysis of Hamiltonian (21).

Neglecting μ in (20) and μy^2 in (21) (which is justified for large \hat{L} and σr , so that $\frac{\mu}{\sigma r} \ll 1$) and varying over μ in (21) one obtains

$$H_{as} = 2\sqrt{p_r^2 + m^2} + \frac{\sigma r}{y} \arcsin y, \quad (22)$$

so that the second term on the r.h.s can be viewed as an effective potential, and we would like to emphasize that this potential is non-trivially l -dependent.

In the general case one has a μ -dependent Hamiltonian (18) with the “potential” $U(\mu, r)$,

$$U(\mu, r) = \frac{\sigma r}{y} \arcsin y + \mu y^2. \quad (23)$$

A simplifying approximation can be used at this step, namely the standard WKB procedure can be applied to the Hamiltonian

$$H = \frac{p_r^2 + m^2}{\mu_0} + \mu_0 + U(\mu_0, r), \quad (24)$$

which slightly differs from the exact Hamiltonian (21) as it treats μ_0 as a variational parameter not depending on τ . We find eigenvalues $M(\mu_0, \hat{L}, n)$ and minimize them with respect to μ_0 to obtain the spectrum $M(\mu_0^*(\hat{L}, n), \hat{L}, n)$, where $\mu_0^*(\hat{L}, n)$ being the extremal value of μ_0 .

To check the accuracy of such a procedure for the eigenvalues two Hamiltonians were considered:

$$H_1 = 2\sqrt{p_r^2 + m^2} + \sigma r, \quad (25)$$

$$H_2 = \frac{p_r^2 + m^2}{\mu_0} + \mu_0 + \sigma r, \quad \mu_0 \text{ varied}, \quad (26)$$

where H_1 is obtained from H_2 in the limit when $\mu_0 \rightarrow \mu(\tau)$.

The results are listed in Table 3. One can see that the accuracy of variational procedure (26) is better than 5% and it is reasonable even for m tending to zero.

n	0	1	2	3	4	5
$M_n^{(1)}$	1.475	2.254	2.825	3.299	3.713	4.085
$M_n^{(2)}$	1.412	2.106	2.634	3.073	3.457	3.803

Table 3: Comparison of the WKB spectrum of Hamiltonian (26) $M_n^{(1)}$ with the exact spectrum of Hamiltonian (25) $M_n^{(2)}$ for $m = 0$ and $\sigma = 0.2 \text{ GeV}^2$.

As a next step we use the standard WKB method to find the spectrum of Hamiltonian (24). To this end we write the Bohr–Sommerfeld condition as

$$\int_{r_-}^{r_+} p_r(r) dr = \pi \left(n + \frac{1}{2} \right), \quad (27)$$

with

$$p_r(r) = \sqrt{\mu_0(M - \mu_0 - U(\mu_0, r)) - m^2}. \quad (28)$$

The eigenvalues $M(\mu_0, \hat{L}, n)$ were found numerically from (27) and the minimization procedure was then used with respect to μ_0 . Results for M_{nl} are given in Table 4 and depicted in Fig.1 demonstrating very nearly straight lines with approximately string slope $(2\pi\sigma)^{-1}$ in l and as twice as smaller slope in n .

$n \ l$	1	2	3	4	5
0	1.865	2.200	2.481	2.729	2.956
1	2.562	2.832	3.068	3.281	3.480
2	3.091	3.329	3.540	3.733	3.913
3	3.535	3.753	3.947	4.125	4.290
4	3.925	4.128	4.309	4.476	4.629
5	4.278	4.469	4.638	4.797	4.939

Table 4: Quasiclassical spectrum of Hamiltonian (18) for $m = 0$ and $\sigma = 0.2 \text{ GeV}^2$.

Let us give a little comment concerning effective potential $U(\mu_0, r)$. Its behaviour at large and small distances can be extracted analytically from Hamiltonian (18) and coincides with that of the Salpeter: the centrifugal barrier at small r and linear growth at large r .

Meanwhile in the region of intermediate values of r this potential differs from what one would have in the Salpeter equation and it is just this region which is important to obtain the correct Regge slope. The form of the effective potential is depicted in Fig.2 for several different angular momenta l . In case of $l = 0$ the effective potential equals to σr for all values of r .

4 Conclusion

We have shown that the proper account of the string dynamics leads to practically linear Regge trajectories, shown in Fig.1, with the slope numerically close to the conventional $(2\pi\sigma)^{-1}$. The exact form of the effective potential incorporating the string rotation as well as the quark radial motion was found numerically and shown in Fig.2.

To make contact with experimental data on meson masses one should specify corrections ΔM_p^2 , ΔM_s^2 in (1), or in the case of the Hamiltonian formalism, one should add to Hamiltonian (21) the colour Coulomb term V_C and spin-dependent interaction.

Treating the latter as perturbation one finds, *e.g.* for ρ meson, a negative shift of the mass due to V_C of about 160 MeV and positive correction of about 40 MeV due to hyperfine term ΔH_{ss} . Taking this into account one obtains for ρ meson ($l = 0$) the mass about 1.6 GeV. It is clear from Fig.1 that these corrections practically do not violate the linearity of Regge trajectories as ρ meson lies on the continuation of the leading theoretical trajectory in l (see dashed line attached to the trajectory with $n = 0$). In this way starting from QCD and making one assumption of the area law for the Wilson loop we obtain linear Regge trajectories for light quark mesons with the string slope.

In this discussion quark spin effects have been taken into account perturbatively, which is a reasonable approximation for the ρ trajectory, but unacceptable for pions and kaons, since for the latter one needs the full implementation of the chiral dynamics.

The progress in this direction was achieved in recent papers of one of the authors (Yu.S.) [10], where an effective Dirac equation for the quark moving in the field of an infinitely heavy antiquark source was derived, and it was shown that solutions display the properties of confinement and chiral symmetry breaking. The nonrelativistic limit of the resulting interaction lead to the conventional result for the confining term and to the spin-orbit interaction in agreement with the standard Eichten-Feinberg-Gromes results [13]. Therefore pionic trajectories should be considered in this new formalism.

There is yet another question unanswered by our paper (and to our knowledge by all other existing papers) — the intercept of Regge trajectories $L_0 \equiv L (M^2 = 0)$. Theoretical intercept for the leading trajectory in j (see Fig.1 and the caption to it) is around -0.5, whereas it is +0.5 for the experimental ρ trajectory also shown in Fig.1. The customary way in the potential models is to add to the Hamiltonian a large negative constant $|C_0| \sim 1$ GeV to reproduce the intercept, but this would obviously violate the linearity of Regge trajectories. Therefore one expects that QCD provides a negative constant ΔM_p^2 in (1) but not in Hamiltonian (18).

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References

- [1] D.R.Stanley and D.Robson, Phys.Lett.**B45** (1980) 235
P.Cea, G.Nardulli and G.Preparata, Z.Phys. **C16** (1982) 135, Phys. Lett. **B115** (1982) 310
J.Carlson, *et.al.* Phys.Rev. **D27** (1983) 233
N.Isgur and S.Godfrey, Phys.Rev. **D32** (1985) 189
- [2] J.L.Basdevant and S.Boukraa, Z.Phys. **C28** (1983) 413
- [3] G.F.Chew, Rev.Mod.Phys. **34** (1962) 394
I.Yu.Kobzarev, B.V. Martemyanov, M.G.Schepkin, UFN **162** (1992) 1 (in Russian)
M.G.Olsson Phys.Rev.**D55** (1997) 5479
B.M.Barbashov and V.V.Nesterenko, *Relativistic string model in hadron physics*, Energoatomizdat, Moscow, 1987 (in Russian)
- [4] Yu.A.Simonov, in Proceedings of Hadron'93 (Como, 21-25 June 1993), ed. T.Bressani, A.Felicielo, G.Preparata and P.G.Ratcliffe;
Yu.S.Kalashnikova and Yu.B.Yufryakov, Phys. Lett. **B359** (1995) 175; Yad.Fiz. **60** (1997) 374
- [5] A.Yu.Dubin, A.B.Kaidalov and Yu.A.Simonov, Phys.Lett. **B323** (1994) 41; Yad.Fiz. **56** (1993) 213
- [6] Dan La Course and M.G.Olsson, Phys.Rev. **D39** (1989) 1751
C.Olsson and M.G.Olsson, MAP/PH/76/(1993)
- [7] V.D.Mur, B.M.Karnakov and V.S.Popov, in preparation, to be submitted to ZhETP
- [8] V.D.Mur, V.S.Popov, Yu.A.Simonov and V.P.Yurov, JETP **75** (1994) 1
- [9] M.S.Marinov and V.S.Popov, JETP **67** (1974) 1250
V.L.Eletsy, V.D.Mur, P.S.Popov and D.N.Voskresensky, JETP **76** (1979) 431;
Phys.Lett. **B80** (1978) 68
- [10] Yu.A.Simonov, Yad. Fiz. **60** (1997) 2252
- [11] L.Brink, P.Di Vecchia, P.Howe, Nucl.Phys. **B118** (1977) 76
- [12] Yu.S.Kalashnikova and A.V.Nefediev, Phys.At.Nucl. **60** (1997) 1389
Yu.S.Kalashnikova and A.V.Nefediev, Phys.At.Nucl. **61** (1998) 785
- [13] N.Brambilla and A.Vairo, Phys.Lett. **B407** (1997) 167
Yu.S.Kalashnikova and A.V.Nefediev, Phys.Lett. **B414** (1997) 149

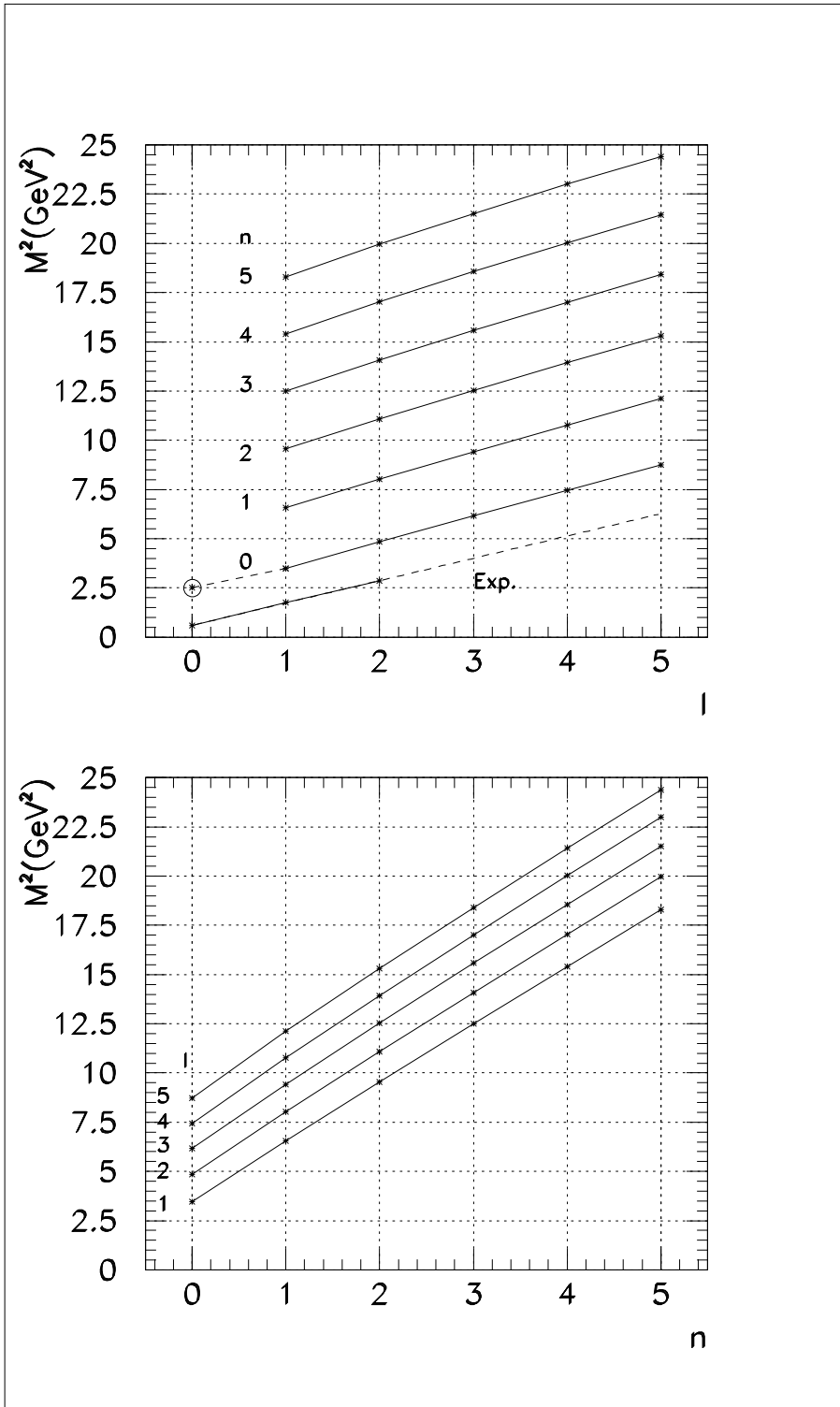


Figure 1: Quasiclassical spectrum of Hamiltonian (18) for $m = 0$ and $\sigma = 0.2 \text{ GeV}^2$. The leading experimental Regge trajectory in angular momentum l is given in the upper plot for comparison. Note that as soon as $l = j - 1$ for the given experimental trajectory, then it lies somewhat higher when plotted in l than when plotted in j . Theoretical prediction for the ρ -meson mass, $M_\rho^2 \approx 2.5 \text{ GeV}^2$, with colour Coulomb interaction and spin effects included, is shown not to violate the straight-line behaviour of the leading theoretical trajectory.

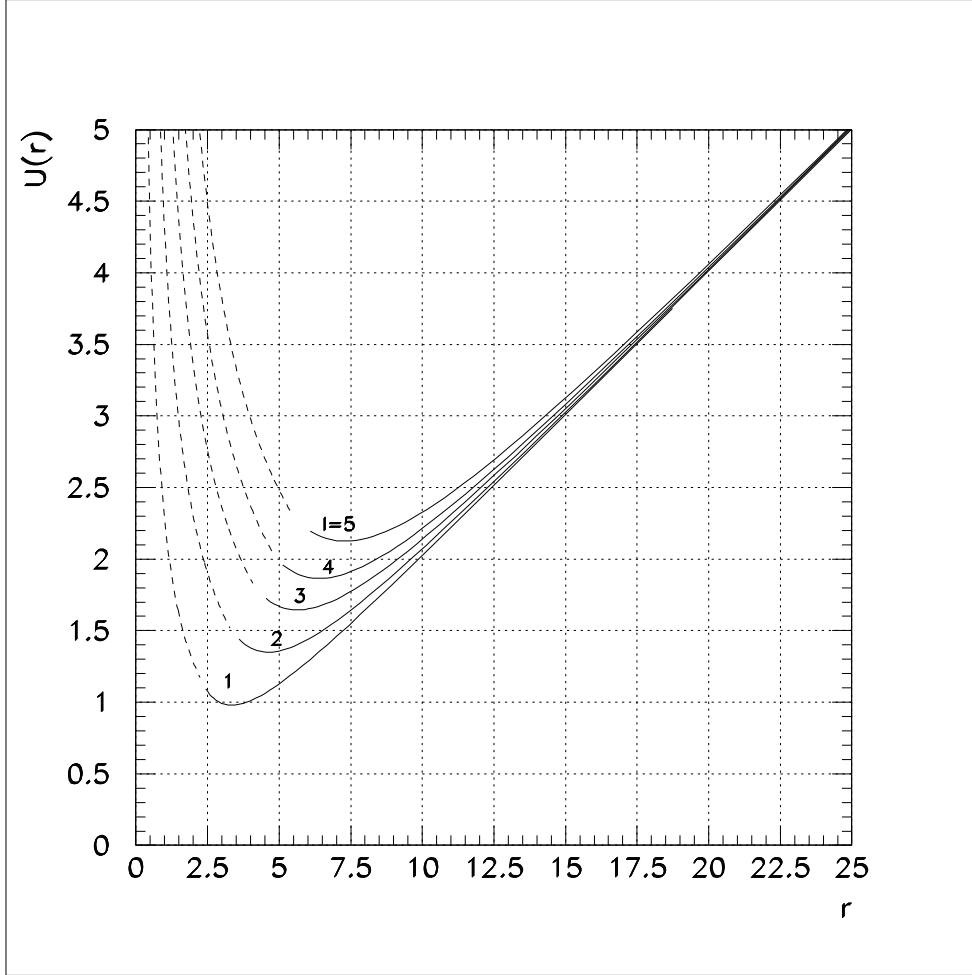


Figure 2: Effective potential incorporating the string rotation as well as the quark radial motion for $\sigma = 0.2 \text{ GeV}^2$. Dashed lines give matching with the centrifugal barrier at small r , whereas for all $l > 0$ the effective potentials have the same asymptotics σr at large r . For $l = 0$ the effective potential coincides with σr for all values of r .